Pavel V. Shevchenko
Quantitative Risk Management

CSIRO Mathematical and Information Sciences

www.csiro.au

Quantitative modelling of financial risks

Australia-Japan Workshop on Data Science
Keio University, 24-27 March 2009

Australian Government
豪日交流基金
Australia-Japan FOUNDATION

CSIRO

Financial Risk in QRM

CMIS Quantitative Risk Management (QRM) group:

- 20 researchers www.cmis.csiro.au/QRM
- ***Financial risk*, *infrastructure, security, air-transport***
- Activities/modes of engagement: ***research, consulting, model development/validation, software development***

Financial risk team in QRM: 6 researchers

Industry: banks, insurance/electricity companies, hedge funds

Beneficiaries of our work in risk management:

- Financial institutions and their shareholders (risk reduction)
- Stability of banking/insurance systems
- Science and CMIS (reputation, external earnings, IP)

Our mission – To conduct research in financial risk quantification, measurement and calculation; To develop innovative quantitative methods and tools in financial engineering

Track Records in Financial Risk since 1999

External Projects

- **Derivative pricing:** option pricing software ***Reditus/Fenics*** since 1999
- **Operational Risk:** risk engines for 2 major banks in Australia, validation, model development and software development projects
- **Market Risk:** validation and model development projects
- **Credit Risk:** validation and model development projects
- **Insurance-Underwriting risk:** consulting projects
- **Electricity:** consulting projects
- **Commodities/interest rates:** consulting projects

Strategic research projects: operational/credit/market risks, option pricing, commodity/interest rate modelling, portfolio allocation

Collaborators: Swiss Federal Institute of Technology (ETH Zurich), Vienna Uni of Technology, Monash Uni, Cambridge Uni, UNSW, UTS, Macquarie Uni, Statistical Research Associates NZ.

Financial risk and other risk areas

Financial Risk

- Market Risk
- Credit Risk
- Operational Risk
- Underwriting risk
- Derivative pricing
- Interest Rates
- Trading strategies
- Portfolio Management
- Commodity/Energy
- Carbon Trading
- Liquidity risk

Links

- Extreme Value models
- Expert Elicitation
- Combining expert&data
- Bayesian methods
- Dependence modelling
- Numerical methods (PDE, MCMC, MC)
- Time series analysis
- State-space models
- High performance computing
- Optimisation

Other risk areas

- Air transport
- Ecology
- Environment
- Infrastructure
- Security
- Weather/Climate
- Health

Regulatory Requirements for banking industry

- Regulatory standards in banking industry:

Basel Committee for banking supervision 1974
Basel I, 1988: Credit Risk,
Basel I Amendment 1996: Market Risk (VaR)
Basel II 2001- ongoing: Credit Risk, Operational Risk

- Insurance regulation:

International Association of Insurance Supervisors
Joint Forum on Financial Conglomerates 1996
Solvency II (similar to Basel II for banks)

Basel II

- **Basel II** requires that banks hold adequate capital to protect against Market risk, Credit risk and Operational Risk losses. Three-pillar framework:

  Pillar 1: minimal capital requirements (risk measurement)

  Pillar 2: supervisory review of capital adequacy

  Pillar 3: public disclosure

  Advanced Measurement Approaches allow internal models for calculations of capital as a large quantile of loss distribution.

  Operational Risk is new risk category
- In Australia, the national regulator APRA is now applying the Basel II requirements.
- **Solvency 2** for insurance industry

Financial Risks – quantifying uncertainties

Risks are modelled by random variables mapping unforeseen future states of the world into values representing profits and losses.

Market Risk: modelling of losses incurred on a trading book due market movements (interest rates, exchange rates, etc)

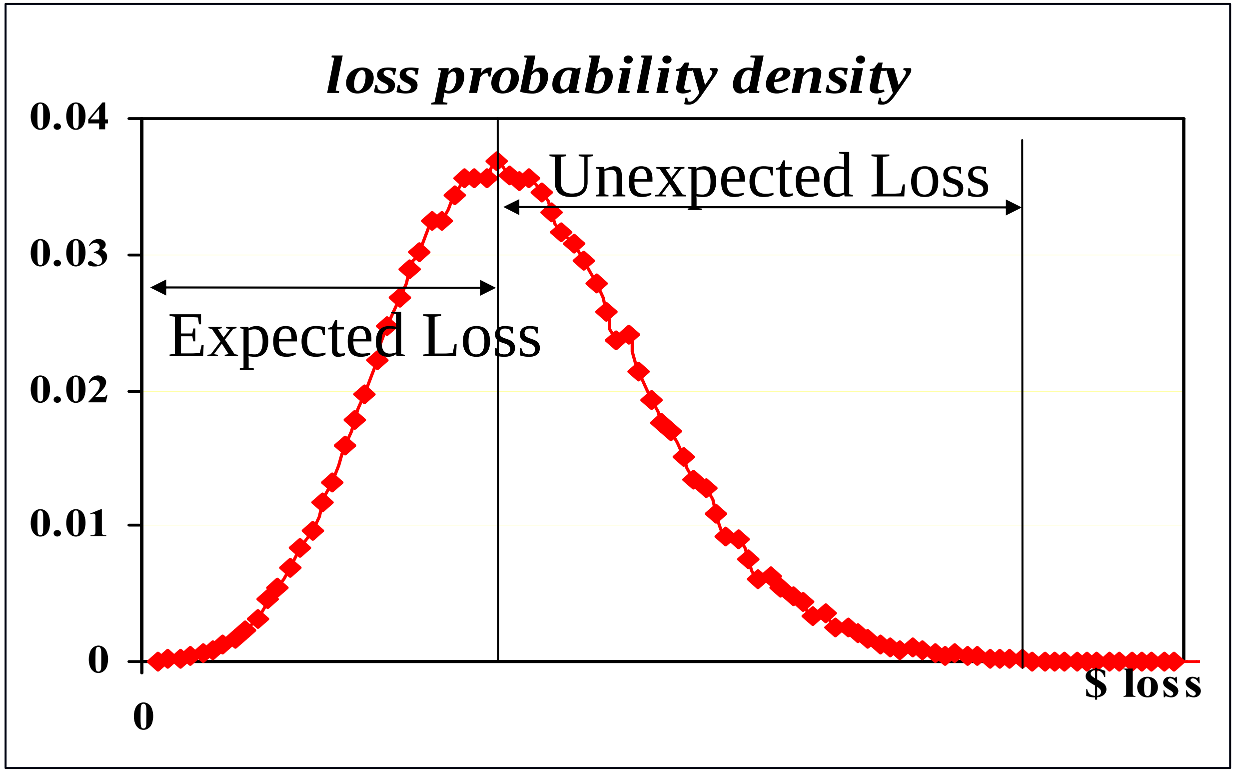
Credit risk: modelling of losses from credit downgrades and defaults (e.g. loan defaults)

Operational Risk: modelling of losses due to failed internal processes, people or external events (e.g. human errors, IT failures, natural disasters, etc)

Quantifying the uncertainties: process uncertainty, parameter uncertainty, numerical error, model error

Regulatory Requirements – Risk Measures
Value at Risk (VaR)

- Unexpected loss $= VaR_q -$ expected loss
- Risk measure : $VaR_q = \inf\{x : \Pr[Loss > x] \le 1 - q\}$
- Operational Risk : $q = 0.999$ over 1 year
- Credit Risk : over 1year
- Market Risk : $q = 0.99$ over 10days



Market, Credit and Operational Risks

- Market risk : portfolio of K financial contracts

$$\Pi_t = \sum_{k=1}^{K} Q_k(\mathbf{S}_t); \mathbf{S}_t = (S_t^{(1)},\ldots,S_t^{(M)}) \text{ stocks, commodities, FX, etc}$$

$$X_t = \mu_0 + \sigma_t Z_t;\ X_t = (S_t - S_{t-1})/S_{t-1}, \sigma_t^2 = \alpha_0 + \alpha_1 (X_{t-1} - \mu_0)^2 + \beta\sigma_{t-1}^2$$

- Credit risk : portfolio of M obligors : $L = \sum_{i=1}^{M} X_i \times I_i$

X_i is loss given default; $I_i = 1$ or 0; $\Pr[I_i = 1] = p$ is prob. of default

- Operational risk : $L = \sum_{j=1}^{56} Z_j; \quad Z_j = \sum_{i=1}^{N_j} X_i^{(j)}$

Z_j is the annual loss due to risk j

N_j and $X_1^{(j)},\ldots,X_{N_j}^{(j)}$ are the annual frequency and severities for risk j

Basel II, Operational Risk

•Basel II defined Operational Risk as: *the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events*. This definition includes legal risk but excludes strategic and reputational risk.

•Losses can be extreme, e.g.
1995: Barings Bank (loss GBP1.3billion)
1996: Sumitomo Corporation (loss US$2.6billion)
2001: September 11
2001: Enron, USD 2.2b (largest US bankruptcy so far)
2002: Allied Irish, £450m
2004: National Australia bank (A$360m)
2008: Société Générale (Euro 4.9billion)

Operational Risk Basel II Business Lines/Event Types

Basel II Business Lines (BL)

- BL(1) - Corporate finance
- BL(2) - Trading & Sales
- BL(3) - Retail banking
- BL(4) - Commercial banking
- BL(5) - Payment & Settlement
- BL(6) - Agency Services
- BL(7) - Asset management
- BL(8) - Retail brokerage

Basel II Event Type (ET)

- ET(1) - Internal fraud:
- ET(2) - External fraud:
- ET(3) - Employment practices and workplace safety:
- ET(4) - Clients, products and business practices
- ET(5) - Damage to physical assets:
- ET(6) - Business disruption and system failures:
- ET(7) - Execution, delivery and process management:

Loss Data Collection Exercise 2007 Japan - Annualized data:

Number of events(100%=947.7) / Total Loss Amount (100%=JPY 22,650mln)

	ET(1)	**ET(2)**	**ET(3)**	**ET(4)**	**ET(5)**	**ET(6)**	**ET(7)**	**Total**
BL(1)	0.00% 0.00%	0.01% 0.00%	0.01% 0.00%	0.11% 0.09%	0.00% 0.00%	0.07% 0.04%	0.28% 0.18%	0.5% 0.3%
BL(2)	0.01% 0.00%	0.00% 0.00%	0.04% 0.04%	0.22% 0.09%	0.00% 0.00%	0.26% 0.00%	**4.12%** **25.03%**	**4.7%** **25.2%**
BL(3)	0.54% **1.24%**	**35.73%** **6.36%**	0.14% 0.18%	**3.62%** **4.77%**	1.11% 0.26%	2.83% 0.26%	**13.21%** **8.43%**	**57.2%** **21.5%**
BL(4)	0.29% 0.13%	0.64% **1.99%**	**1.16%** 0.71%	**2.02%** **18.32%**	0.69% **4.19%**	**5.87%** **3.22%**	**15.05%** **16.87%**	**25.7%** **45.4%**
BL(5)	0.00% 0.00%	0.00% 0.00%	0.00% 0.00%	0.00% 0.00%	0.00% 0.00%	0.38% 0.04%	0.23% 0.00%	0.6% 0.1%
BL(6)	0.00% 0.00%	0.00% 0.00%	0.01% 0.00%	0.92% 0.62%	0.01% 0.00%	**1.19%** 0.22%	**3.14%** **3.00%**	**5.3%** **3.8%**
BL(7)	0.00% 0.00%	0.00% 0.00%	0.1% 0.04%	0.48% 0.49%	0.00% 0.00%	0.05% 0.00%	**1.50%** **1.02%**	**2.1%** **1.5%**
BL(8)	0.84% **1.32%**	0.00% 0.00%	0.01% 0.00%	**1.40%** 0.53%	0.00% 0.00%	0.18% 0.04%	**1.06%** 0.13%	**3.5%** **2.0%**
Other	0.09% 0.18%	0.11% 0.00%	0.02% 0.00%	0.00% 0.00%	0.13% 0.00%	0.09% 0.00%	0.02% 0.00%	0.4% 0.2%
Total	**1.8%** **2.9%**	**36.5%** **8.3%**	**1.5%** **1.0%**	**8.8%** **24.8%**	**1.9%** **4.5%**	**10.9%** **3.9%**	**38.6%** **54.6%**	100% (no.events) 100% (loss)

Bank of Japan, et al (2007)

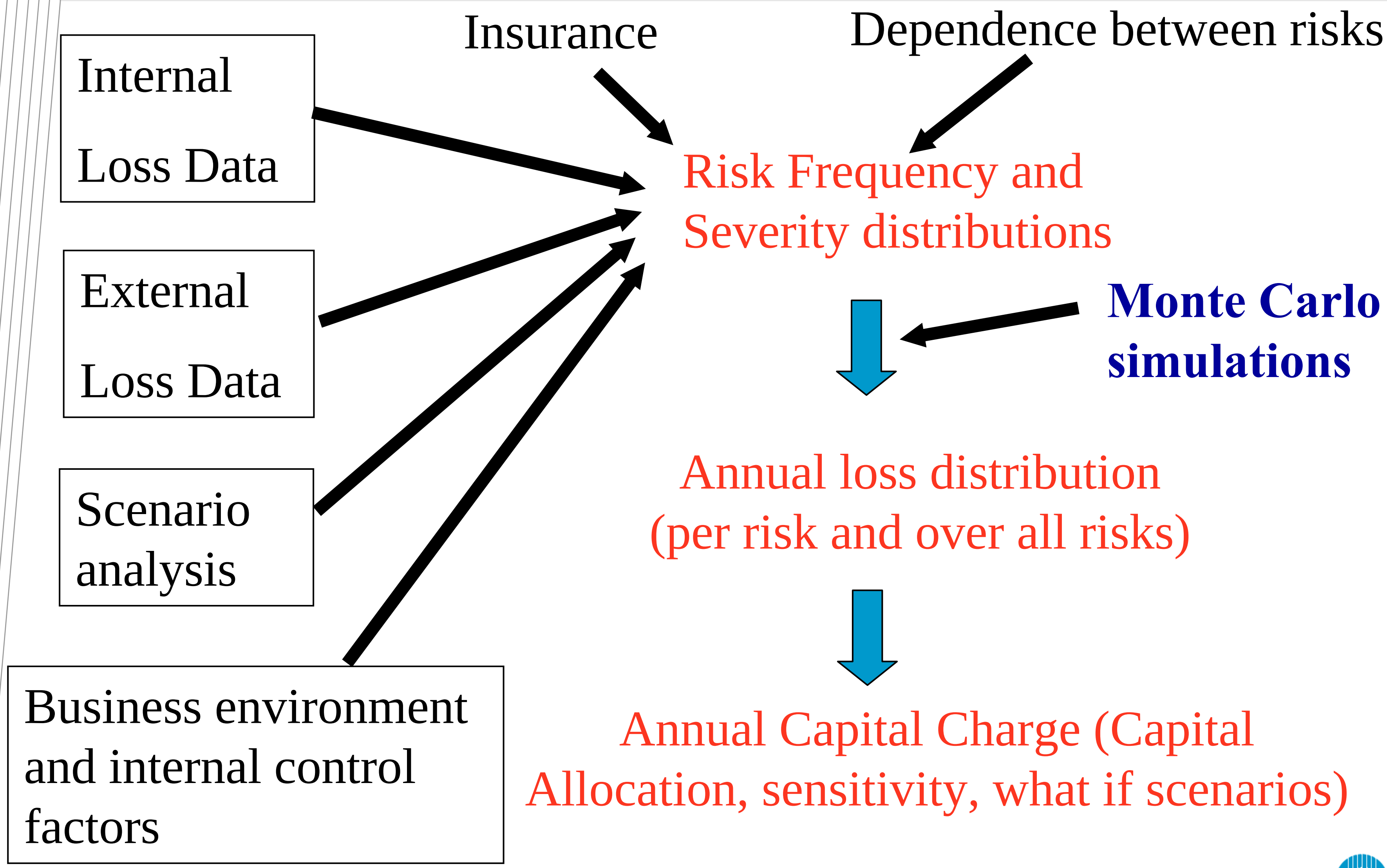
Loss Data Collection Exercise 2004 USA - Annualized data:

Number of events(100%=18371) / Total Loss Amount (100%=USD 8,643mln)

	ET(1)	ET(2)	ET(3)	ET(4)	ET(5)	ET(6)	ET(7)	Other	Fraud	Total
BL(1)	0.01% 0.14%	0.01% 0.00%	0.06% 0.03%	0.08% 0.30%	0.00% 0.00%		0.12% 0.05%	0.03% 0.01%	0.01% 0.00%	0.3% 0.5%
BL(2)	0.02% 0.10%	0.01% **1.17%**	0.17% 0.05%	0.19% **4.29%**	0.03% 0.00%	0.24% 0.06%	**6.55%** **2.76%**		0.05% 0.15%	**7.3%** **8.6%**
BL(3)	**2.29%** 0.42%	**33.85%** **2.75%**	**3.76%** 0.87%	**4.41%** **4.01%**	0.56% 0.1%	0.21% 0.21%	**12.28%** **3.66%**	0.69% 0.06%	**2.10%** 0.26%	**60.1%** **12.3%**
BL(4)	0.05% 0.01%	**2.64%** 0.70%	0.17% 0.03%	0.36% 0.78%	0.01% 0.00%	0.03% 0.00%	**1.38%** 0.28%	0.02% 0.00%	0.44% 0.04%	**5.1%** **1.8%**
BL(5)	0.52% 0.08%	0.44% 0.13%	0.18% 0.02%	0.04% 0.01%	0.01% 0.00%	0.05% 0.02%	**2.99%** 0.28%	0.01% 0.00%	0.23% 0.05%	**4.5%** 0.6%
BL(6)	0.01% 0.02%	0.03% 0.01%	0.04% 0.02%	0.31% 0.06%	0.01% 0.01%	0.14% 0.02%	**4.52%** 0.99%			**5.1%** **1.1%**
BL(7)	0.00% 0.00%	0.26% 0.02%	0.10% 0.02%	0.13% **2.10%**	0.00% 0.00%	0.04% 0.01%	**1.82%** 0.38%		0.09% 0.01%	**2.4%** **2.5%**
BL(8)	0.06% 0.03%	0.10% 0.02%	**1.38%** 0.33%	**3.30%** 0.94%		0.01% 0.00%	**2.20%** 0.25%		0.20% 0.07%	**7.3%** **1.6%**
Other	0.42% 0.1%	**1.66%** 0.3%	**1.75%** 0.34%	0.40% **67.34%**	0.12% **1.28%**	0.02% 0.44%	**3.45%** 0.98%	0.07% 0.05%	0.08% 0.01%	**8.0%** **70.8%**
Total	**3.40%** 0.9%	**39.0%** **5.1%**	**7.6%** **1.7%**	**9.2%** **79.8%**	0.7% **1.4%**	0.7% 0.8%	**35.3%** **9.6%**	0.8% 0.1%	**3.2%** 0.6%	100.0%, 100.0% (US$)

Federal Reserve System, et al (2005)

Operational Risk - Loss Distribution Approach



Operational Risk: Combining internal data, industry data and expert opinions

- BIS (2006), p.152: "*Any operational risk measurement system must have certain key features to meet the supervisory soundness standard set out in this section. These elements must include the use of internal data, relevant external data, scenario analysis and factors reflecting the business environment and internal control systems.*"
- An interview with four industry's top risk executives: Davis, E. (1 September 2006). Theory vs Reality, OpRisk and Compliance: "*[A] big challenge for us is how to mix the internal data with external data; this is something that is still a big problem because I don't think anybody has a solution for that at the moment." Or: "What can we do when we don't have enough data [. . .] How do I use a small amount of data when I can have external data with scenario generation? [. . .] I think it is one of the big challenges for operational risk managers at the moment*".

Ad-hoc procedures for combining internal data, industry data and expert opinions (SA)

- Internal data to estimate frequency and combined sample of internal & external data to estimate severity
- Mixing distributions: $w_1 F_{SA}(x) + w_2 F_{\text{int}}(x) + (1 - w_1 - w_2) F_{ext}(x)$
- Minimum variance: consider two unbiased independent estimates

  $\hat{\theta}_1$ and $\hat{\theta}_2$ for parameter θ, i.e. $E[\hat{\theta}_m] = \theta; \text{var}[\hat{\theta}_m] = \sigma_m^2, m = 1,2.$

  Combined unbiased estimator:

  $$\hat{\theta}_{tot} = w_1 \hat{\theta}_1 + w_2 \hat{\theta}_2, w_1 + w_2 = 1; \quad \text{var}(\hat{\theta}_{tot}) = w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2$$

  Choose weights to minimize $\text{var}(\hat{\theta}_{tot})$: $\hat{w}_1 = \dfrac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}, \hat{w}_2 = \dfrac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$

  Can be easily extended to combine three or more estimators.

  $$\hat{\theta}_{tot} = w_1 \hat{\theta}_1 + \ldots + w_K \hat{\theta}_K, w_1 + \ldots + w_K = 1; \quad \hat{w} = \arg\min[w : \text{var}(\hat{\theta}_{tot})]$$

Combining three data sources: internal&external data and expert opinion via Bayesian inference

Internal data	Expert opinions	Parameters
$\mathbf{X}=(X_1,X_2,...,X_n)$	$\boldsymbol{\upsilon}=(\upsilon_1,\upsilon_2,...,\upsilon_M)$	$\boldsymbol{\theta}=(\theta_1,\theta_2,...,\theta_K)$

$$\pi(\boldsymbol{\theta}\,|\,\mathbf{X},\boldsymbol{\upsilon})\propto h_1(\mathbf{X}\,|\,\boldsymbol{\theta})h_2(\boldsymbol{\upsilon}\,|\,\boldsymbol{\theta})\pi(\boldsymbol{\theta})$$

$\pi(\boldsymbol{\theta})$ - prior distribution is estimated by industry data

$h_1(\mathbf{X}\,|\,\boldsymbol{\theta})$ - likelihood of internal observations

$h_2(\boldsymbol{\upsilon}\,|\,\boldsymbol{\theta})$ - likelihood of expert opinions

$\pi(\boldsymbol{\theta}\,|\,\mathbf{X},\boldsymbol{\upsilon})$ - posterior density (conjugate priors, MCMC methods, Gaussian approximation)

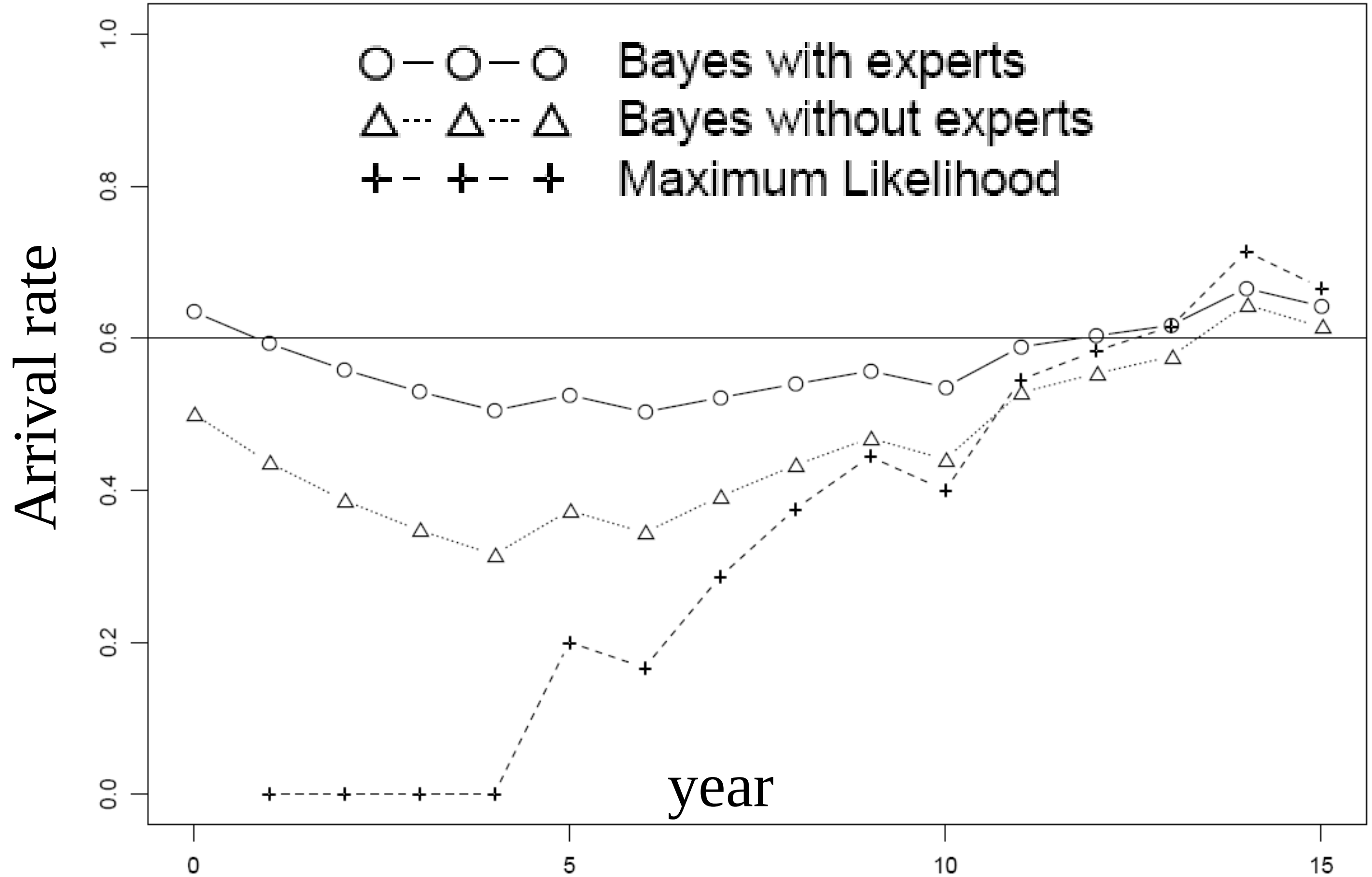
Joint work with ETH Zurich: H. Bühlmann, M.Wüthrich, D.Lambrigger

Combining internal & external data with expert Example: *Poisson-Gamma-Gamma*

Annual counts : $\mathbf{N} = (0,0,0,0,1,0,1,1,1,0,2,1,1,2,0)$ from $Poisson(0.6)$

External data : $E[\lambda] = 0.5,\ \Pr[0.25 \le \lambda \le 0.75] = 2/3 \Rightarrow \alpha \approx 3.41,\ \beta \approx 0.15$

Expert : $\hat{\upsilon} = 0.7, Vco(\upsilon \mid \lambda) = 0.5$



Modelling dependence

•**Basel Committee statement**

"Risk measures for different operational risk estimates must be added for purposes of calculating the regulatory minimum capital requirement. However, the bank may be permitted to use internally determined correlations in operational risk losses across individual operational risk estimates, provided it can demonstrate to the satisfaction of the national supervisor that its systems for determining correlations are sound, implemented with integrity, and take into account the uncertainty surrounding any such correlation estimates (particularly in periods of stress). The bank must validate its correlation assumptions using appropriate quantitative and qualitative techniques." BIS (2006), p.152.

•***Remark***: *Adding capitals implies perfect positive dependence between risks which is too conservative*

Dependence between risks

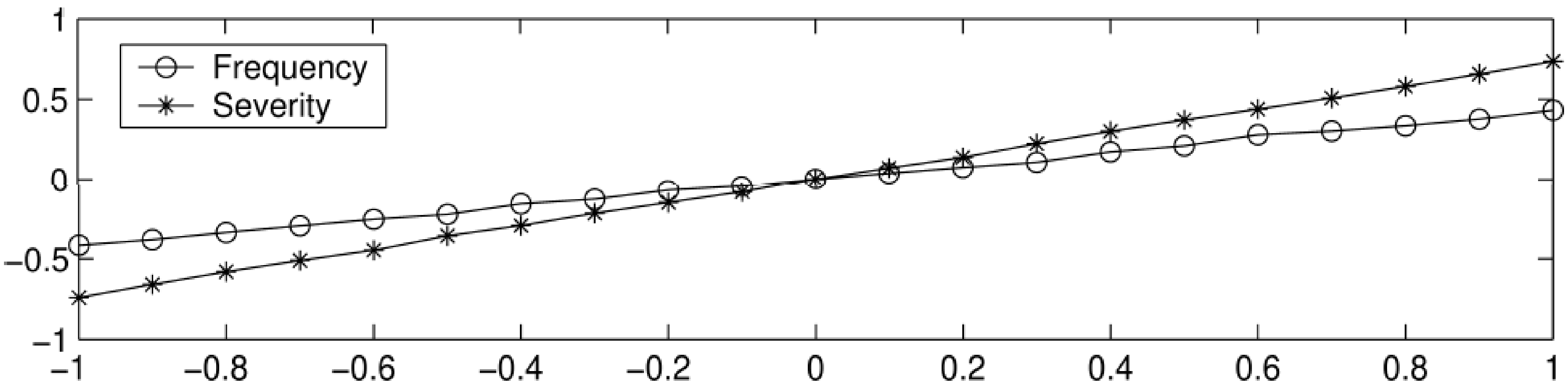
$$total\ annual\ loss: Z = \sum_{k=1}^{K} Z_k = \sum_{i=1}^{N_1} X_i^{(1)} + \sum_{i=1}^{N_2} X_i^{(2)} + \ldots + \sum_{i=1}^{N_K} X_i^{(K)}$$

- Possible diversification in a capital : $\mathrm{C}(Z) \leq \mathrm{C}(Z_1) + \ldots + \mathrm{C}(Z_K)$
- VaR : $VaR_\alpha(Z) = F_Z^{-1}(\alpha) = \min\{z, F_Z(z) \geq \alpha\}$, is not a coherent measure and formally diversification may fail
- Expected shorfall (ES) : $ES_\alpha(Z) = E[Z \mid Z > VaR_\alpha(Z)]$
- Dependence between the frequencies N_i and $N_j, i \neq j$
- Dependence via the common events affecting many risk cells
- Dependence between the severities occured at the same time
- Dependence between the annual losses Z_i and $Z_j, i \neq j$
- Dependence between the risk profiles (stochastic parameters) : calibration via MCMC (Slice Sampler)

Example: dependence between frequency profiles; dependence between severity profiles

$$Z_t^{(i)} = \sum_{s=1}^{N_t^{(i)}} X_s^{(i)}(t) \text{ and } Z_t^{(j)} = \sum_{s=1}^{N_t^{(j)}} X_s^{(j)}(t)$$

- $N_t^{(j)} \sim Poisson(\lambda_t^{(j)}),\ X_s^{(i)}(t) \sim LN(\mu_t^{(j)}, \sigma_t^{(j)})$
- $\lambda_t^{(j)} \sim Gamma(4,10),\ \mu_t^{(j)} \sim N(2,0.1)$ and $\sigma_t^{(j)} \sim Gamma(1,1)$
- dependence between $\lambda_t^{(j)}, \mu_t^{(j)}, \sigma_t^{(j)}$ via copula



Spearman's rank correlation $\rho_S(Z^{(1)}, Z^{(2)})$ vs Gaussian copula parameter ρ.

$(\circ)$ – copula between $\lambda^{(1)}, \lambda^{(2)}$; $(*)$ – copula between $\mu^{(1)}, \mu^{(2)}$.

Parameter Risk (uncertainty of parameters)

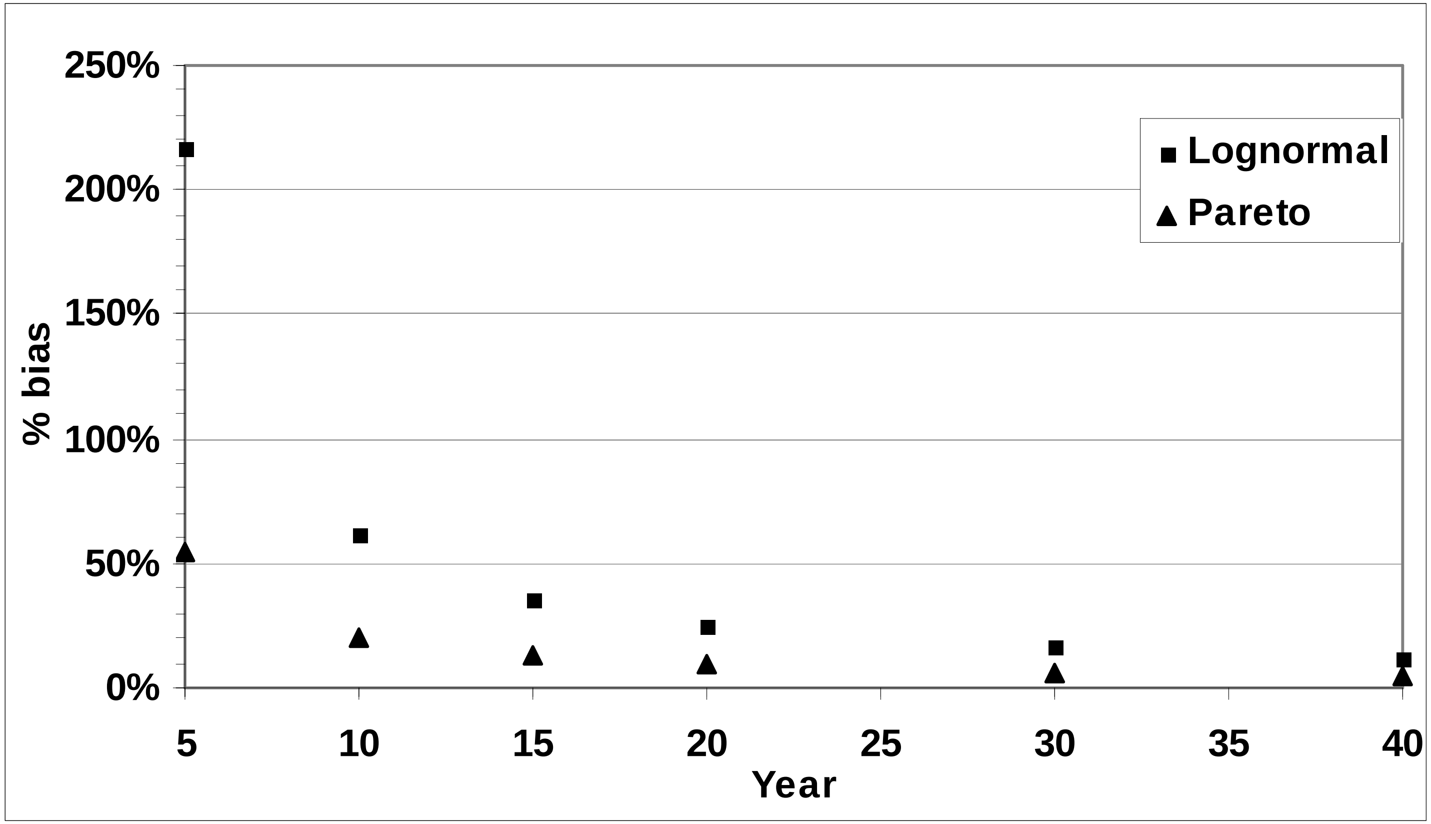
$Z_t = \sum_{i=1}^{N_t} X_i(t)$ – loss in year t;

- $\mathbf{Y} = (\mathbf{X}, \mathbf{N})$ – past observations $t = 1, \ldots, M$
- $\varphi(Z_{M+1} \mid \mathbf{Y}) = \int g(Z_{M+1} \mid \boldsymbol{\theta}) \times \pi(\boldsymbol{\theta} \mid \mathbf{Y}) d\boldsymbol{\theta}$ – full predictive distribution
- $\hat{Q}^B_{0.999}$ – 0.999 quantile of $\varphi(Z_{M+1} \mid \mathbf{Y})$
- $\hat{Q}_{0.999}$ – 0.999 quantile of $g(Z_{M+1} \mid \hat{\boldsymbol{\theta}})$; $\hat{\boldsymbol{\theta}}$ is a point estimator, MLE

$bias = E[\hat{Q}^B_{0.999} - \hat{Q}_{0.999}]$

- $\pi(\boldsymbol{\theta} \mid \mathbf{Y}) \propto h(\mathbf{Y} \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta})$, the posterior density

Parameter Risk (uncertainty of parameters)



Relative bias in the 0.999 quantile estimator induced by the parameter uncertainty vs number of observation years. (Lognormal) - losses were simulated from *Poisson*(10) and *LN*(1,2). (Pareto) – losses were simulated from *Poisson*(10) and *Pareto*(2) with *L*=1.

Modelling commodities/interest rates: state-space models

- Measurement Equation: $\vec{F}_t = \vec{A} + \hat{B} \times \vec{X}_t + \vec{e}$
- Transition Equation: $\vec{X}_{t+1} = \vec{M} + \hat{T} \times \vec{X}_t + \vec{\varepsilon}$

- Commodity spot models: e.g. 2-factor long-short

S_t – spot price; ξ_t, χ_t – long/short rates, $F_{t,T}$ – futures prices

$$\ln S_t = \xi_t + \chi_t + h(t)$$

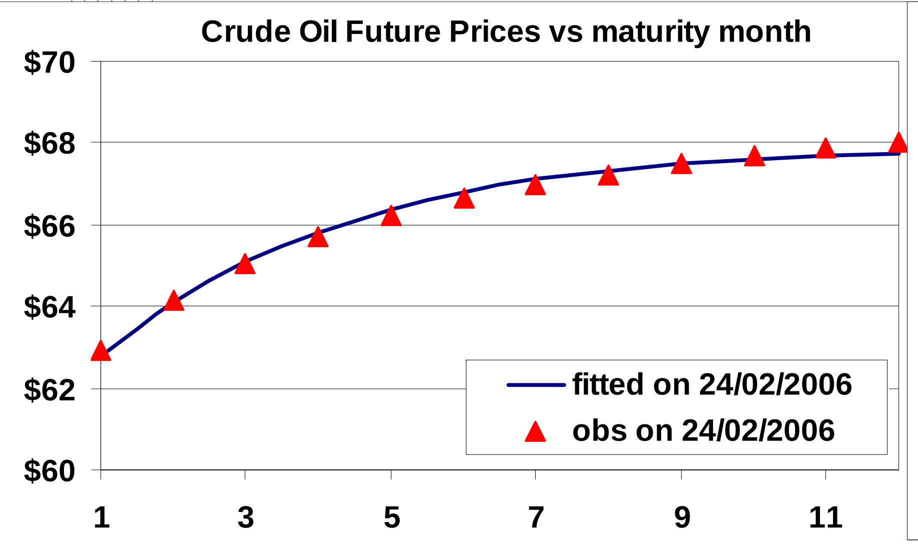
$$d\xi_t = [\mu - \lambda_\xi - \omega \xi_t]dt + \sigma_1 dW_t^{(1)}$$

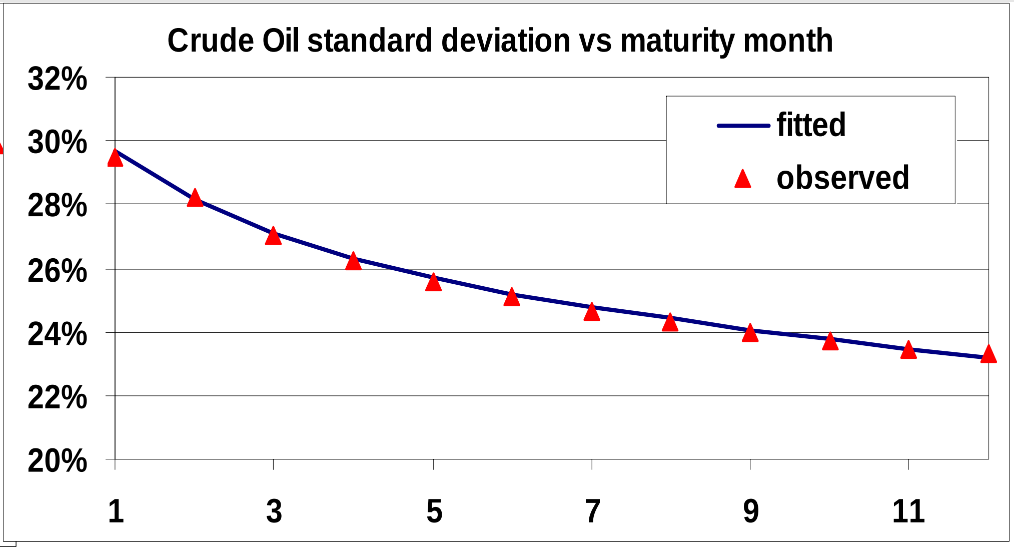
$$d\chi_t = [-\lambda_\chi - \kappa_1 \delta_t]dt + \sigma_2 dW_t^{(2)}; \quad E[dW_t^{(1)} dW_t^{(2)}] = \rho dt$$

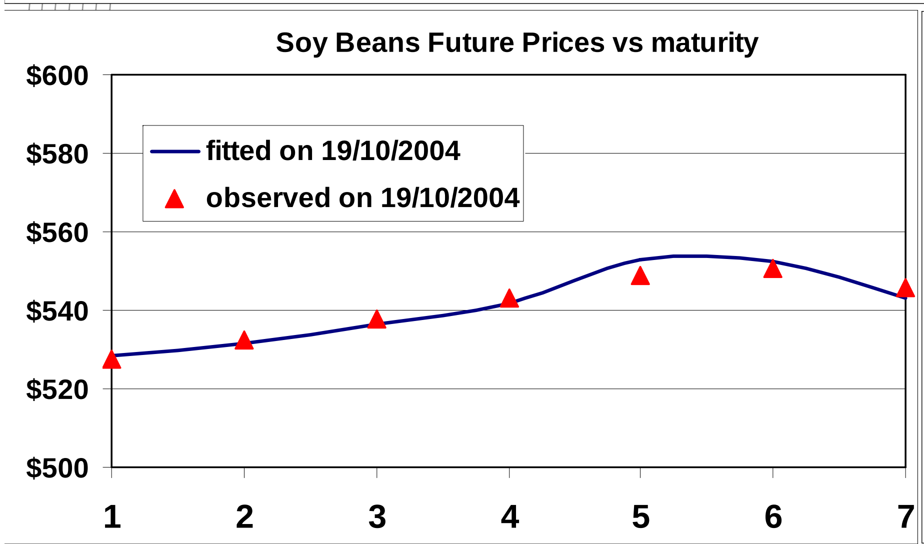
$$F_{t,T} = E[S_T \mid \chi_t, \xi_t] \Rightarrow \ln F_{t,T} = h(T) + A(T-t) + B(T-t)\xi_t + C(T-t)\chi_t$$

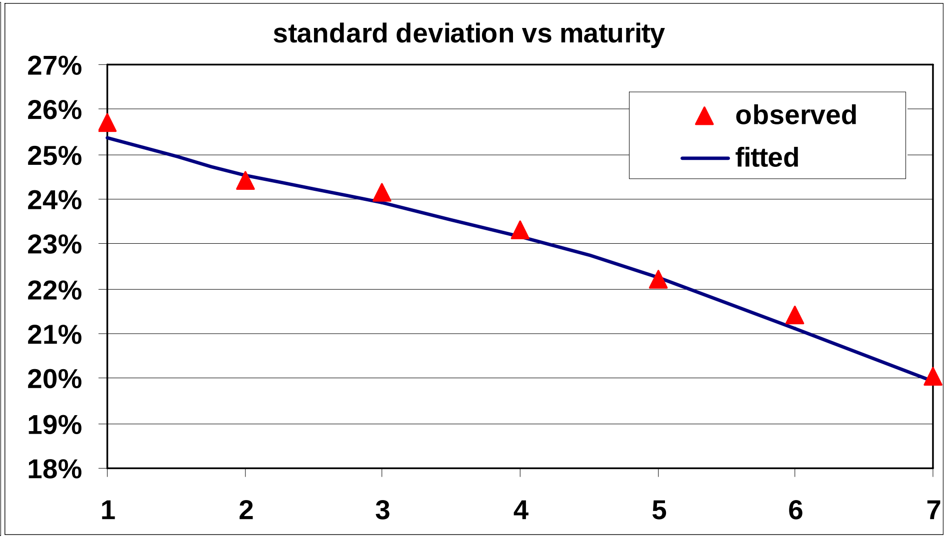
- Non-Gaussian models/fitting option prices – calibration requires *non-linear Kalman filter, particle filter/SMC*

Seasonal and non-seasonal commodities

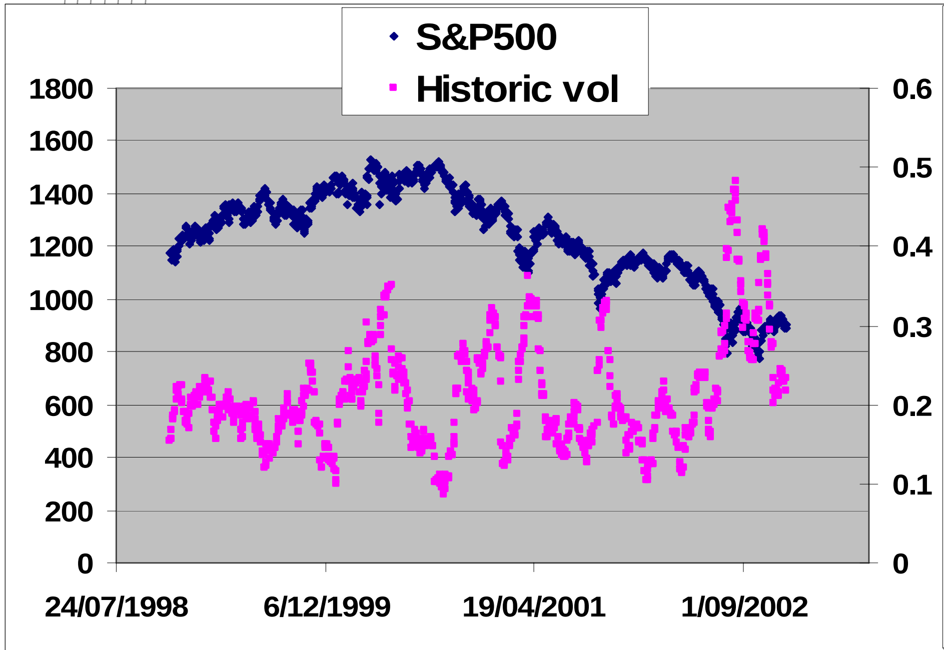


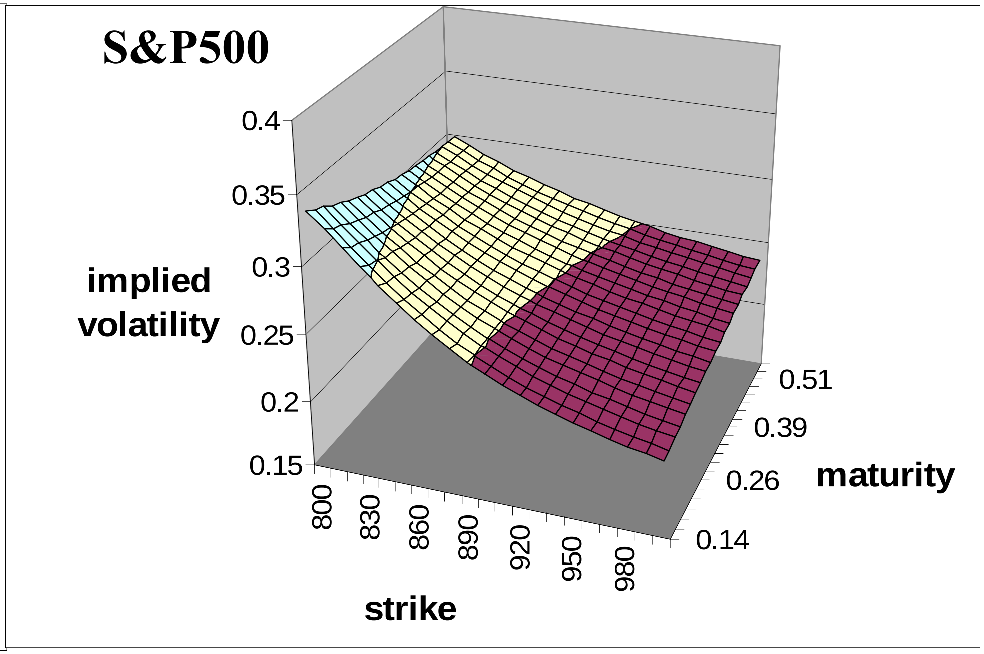






Pricing Exotic Options: Stochastic Volatility, Implied Volatility Smile





Local Volatility models (e.g. Dupire 1993)

Stochastic Volatility and Jump-diffusion models (e.g. Merton 1976, Heston 1993)

Heston stochastic volatility model (1993)

- Risk - neutral process

$$dS_t = S_t(r-q)dt + S_t\sqrt{V_t}dW_t$$

$$dV_t = (\omega - \theta V_t)dt + \xi\sqrt{V_t}dZ_t$$

- Characteristic Function for transition density $p(x,v,t \mid x_0,v_0,t_0),\ X = \ln S$

$$p_k = \int e^{ikx} p(x,v,t \mid x_0,v_0,t_0)dxdv = \exp[C_1(\tau)x_0 + C_2(\tau)v_0 + C_3(\tau)]$$

- Option price via inverse Fourier or PDF approx :

$$Q = \int p(x)h(x)dx = \int p_k h_{-k} dk;\ h_k = \int e^{ikx} h(x)dx;\ p_k = \int e^{ikx} p(x)dx$$

- Call option payoff $h(x) = \max[S-K,0]$,

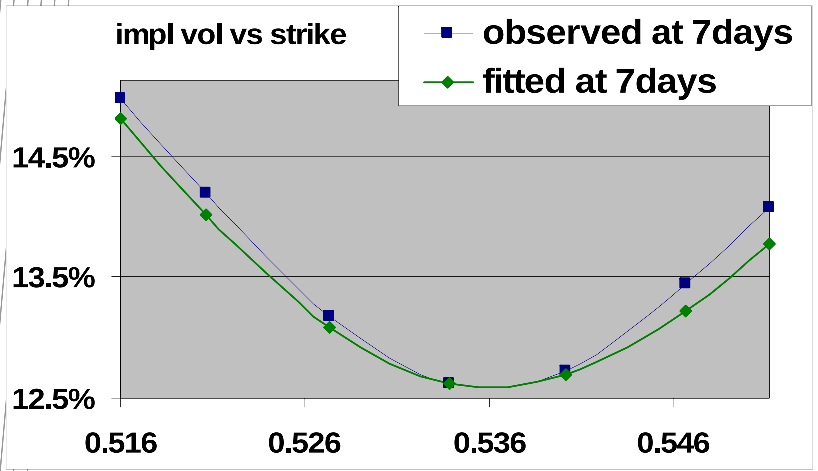
$$h_k = -\frac{K^{ik+1}}{k(k-i)},\quad \mathrm{Im}\, k > 1;\quad C = -\tfrac{1}{2\pi}\int_{-\infty+ia}^{\infty-ia} p_k \frac{K^{-ik+1}}{k(k+i)} dk,\ a < -1$$

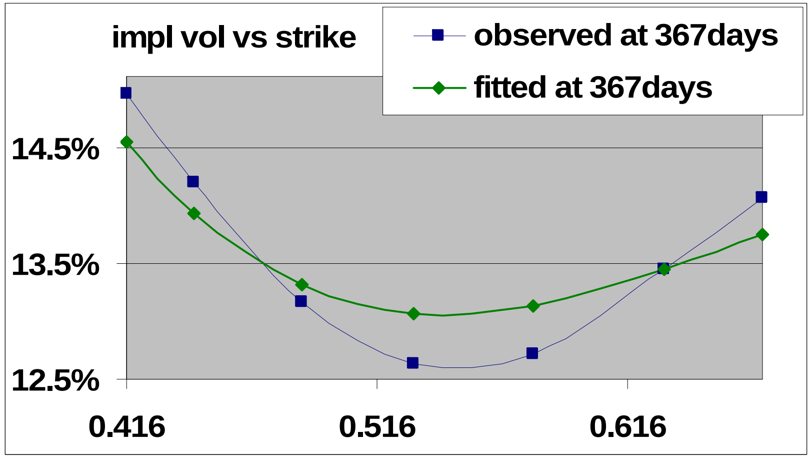
- Put option payoff $h(x) = \max[K-S,0]$

$$h_k = -\frac{K^{ik+1}}{k(k-i)},\quad \mathrm{Im}\, k < 0;\quad P = -\tfrac{1}{2\pi}\int_{-\infty+ia}^{\infty-ia} p_k \frac{K^{-ik+1}}{k(k+i)} dk,\ a > 0$$

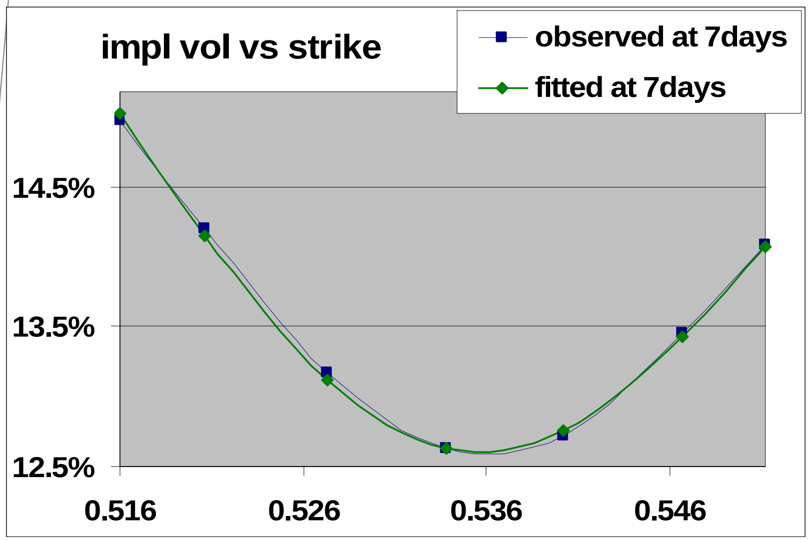
Heston model - Calibration via today's prices

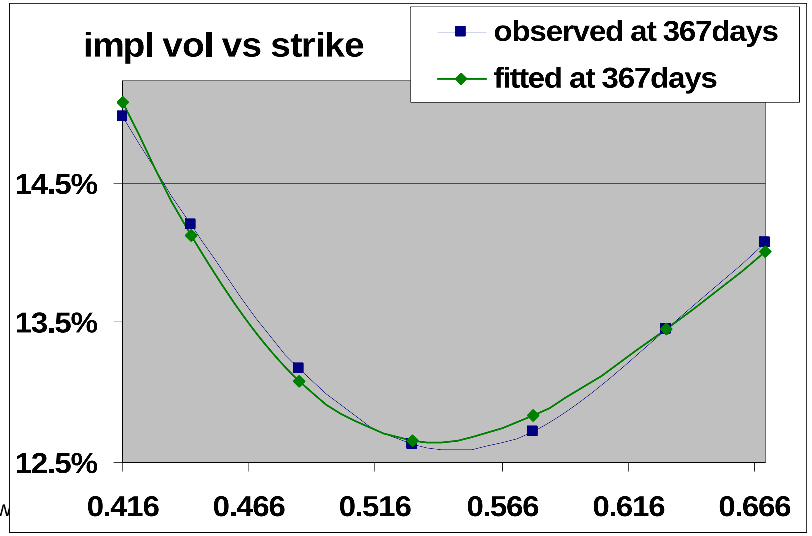
Constant model parameters, US$/AU$





Time-dependent model parameters, US$/AU$





Local volatility model, Dupire 1993

Risk - neutral process : $dS(t)/S(t) = [r(t) - q(t)]dt + \sigma(S,t)dW(t).$

Vanilla option price : $Q(S,t) = \int_0^\infty h(u)\,p(S,t,u,T)du,$

Call payoff : $h(S(T)) = C(S(T),T) = Max[S(T) - K, 0]$

Risk - neutral density : $p(S,t,K,T) = \dfrac{\partial^2 C(S,t,K,T)}{\partial^2 K}$

Fwd. Kolmogorov eq : $\dfrac{\partial p(S,t,u,T)}{\partial T} - \frac{1}{2}\dfrac{\partial^2(\sigma^2(u,T)u^2 p)}{\partial u^2} + \dfrac{\partial(r(T) - q(T))up)}{\partial u} = 0$

Volatility function : $\sigma^2(K,T) = 2\dfrac{\partial C/\partial T + q(T)C + K(r(T) - q(T))\partial C/\partial K}{K^2 \partial^2 C/\partial K^2}$

$$\sigma(K,T) = \sqrt{\frac{2\theta T \dfrac{\partial\theta}{\partial T} + \theta^2 + 2rK\theta T \dfrac{\partial\theta}{\partial K}}{\left[1 + d_1 K\sqrt{T}\dfrac{\partial\theta}{\partial K}\right]^2 + K^2\theta T\left[\dfrac{\partial^2\theta}{\partial K^2} - d_1\left(\dfrac{\partial\theta}{\partial K}\right)^2\sqrt{T}\right]}}.$$

θ is implied vol

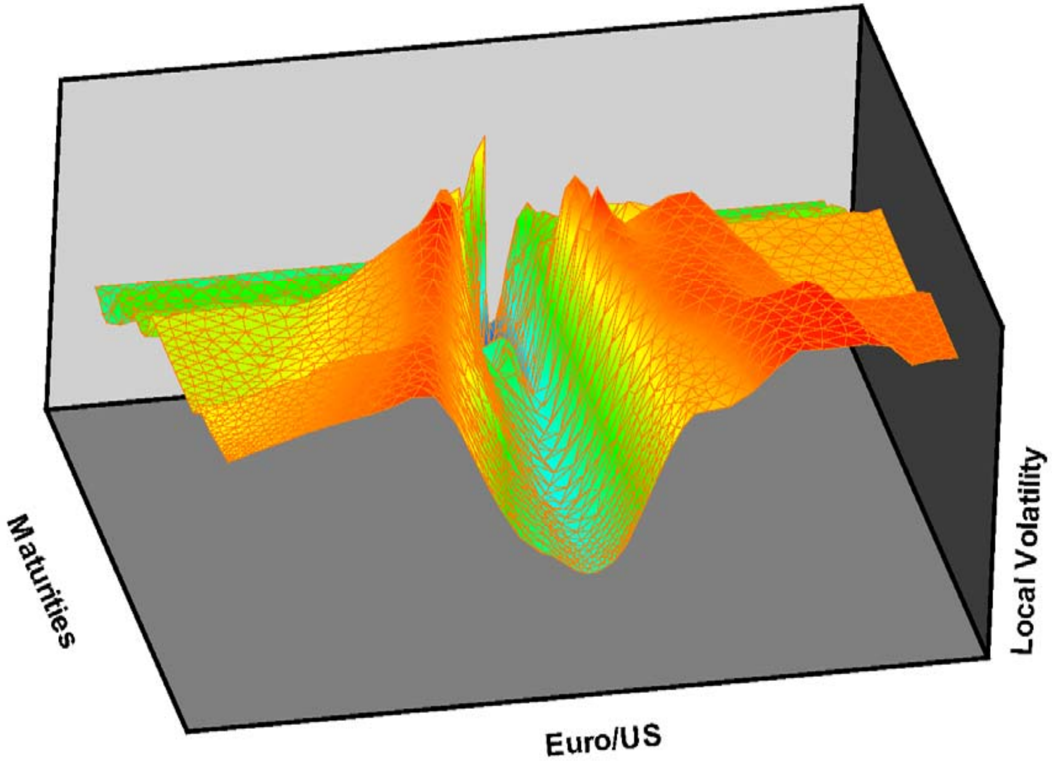
Local volatility model – Exotic Option Pricing

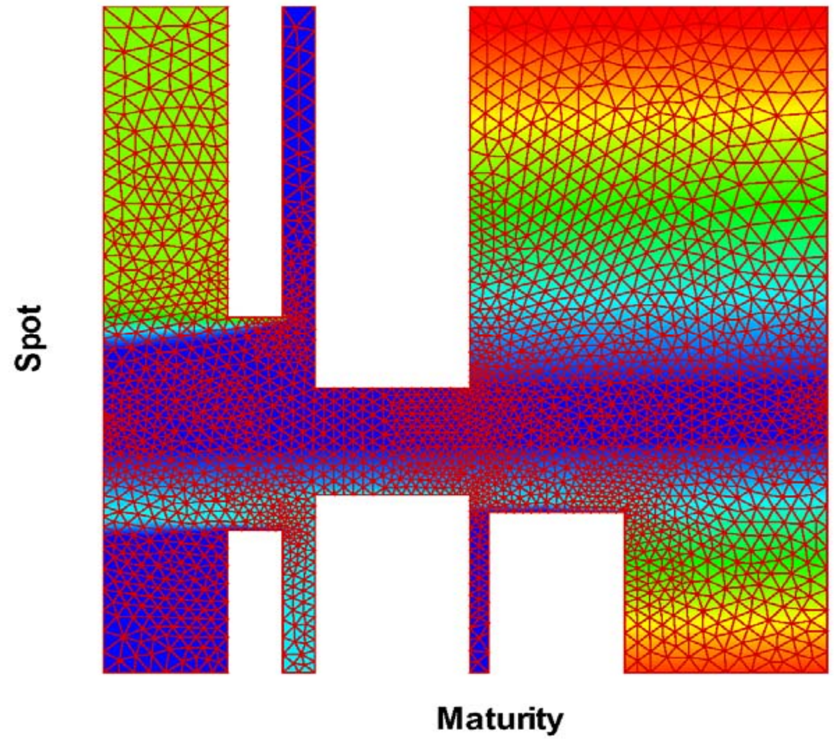
$$\text{Option price}: Q(S_t, t) = E[\text{Payoff}(S_T)]; \text{e.g. call payoff} = \max[S_T - K, 0]$$

$$dS_t / S_t = [r(t) - q(t)]dt + \sigma(S,t)dW(t).$$

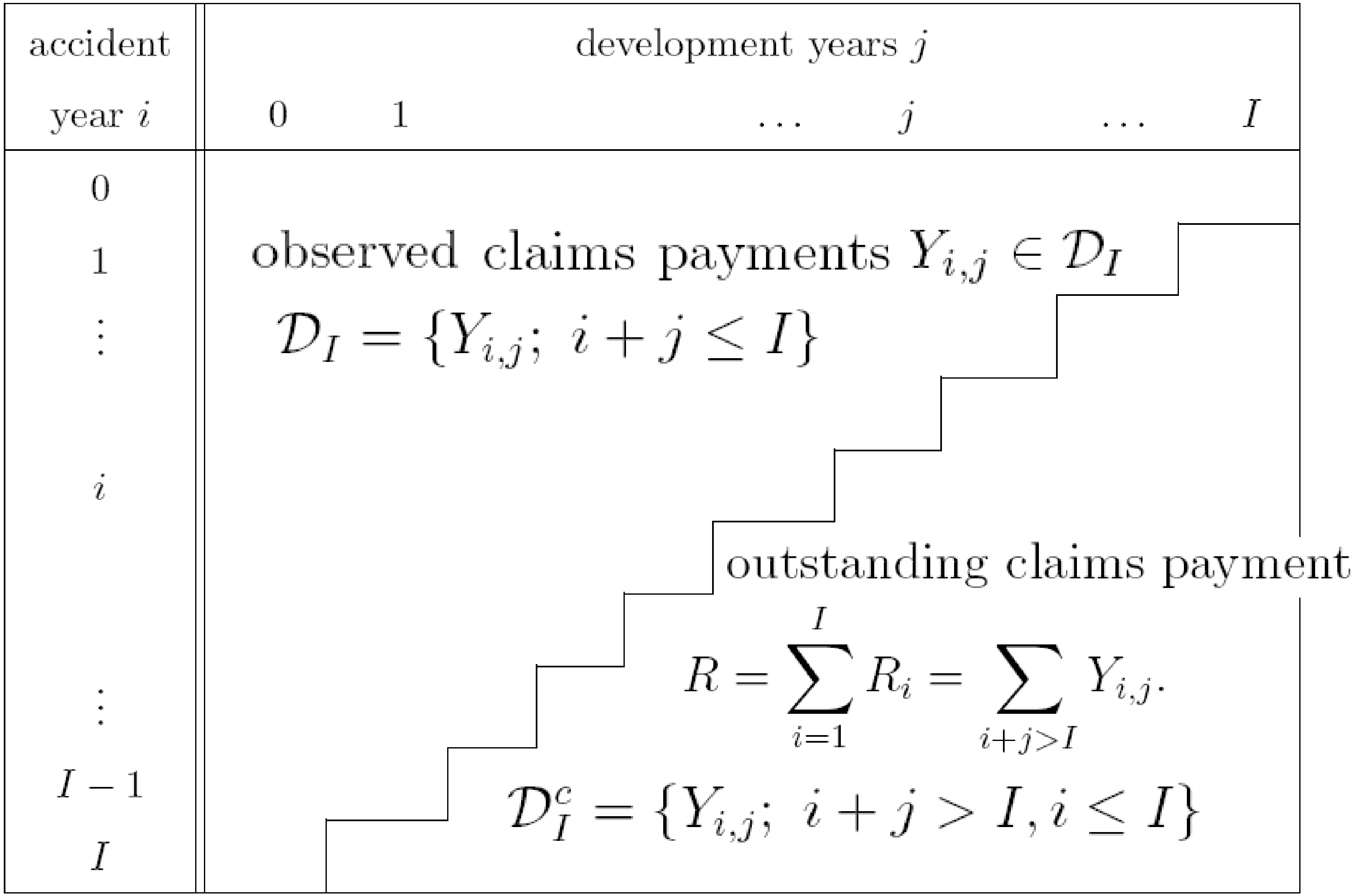
$$\frac{\partial Q(S,t)}{\partial t} + \tfrac{1}{2}\sigma^2(S,t)S^2 \frac{\partial^2 Q(S,t)}{\partial S^2} + (r(t) - q(t))S \frac{\partial Q(S,t)}{\partial S} = r(t)Q(S,t)$$

Numerical PDE methods: finite difference, finite element





Claims Reserving (non-life insurance), solvency requirements, claims development triangle



Joint work with M.Wüthrich (ETH Zurich) and G.Peters (UNSW/CSIRO)

$\widehat{R}$ - predictor for R and estimator for $E[R|\mathcal{D}_I]$

$$R = \sum_{i=1}^{I} R_i = \sum_{i+j>I} Y_{i,j} \qquad E[R|\mathcal{D}_I] = \sum_{i=1}^{I} E[R_i|\mathcal{D}_I]$$

$$\mathrm{msep}_{R|\mathcal{D}_I}\left(\widehat{R}\right) = E\left[\left(R-\widehat{R}\right)^2 \middle| \mathcal{D}_I\right]$$

Mean Square Error of Prediction

$$\mathrm{msep}_{R|\mathcal{D}_I}\left(\widehat{R}\right) = \mathrm{Var}(R|\mathcal{D}_I) + \left(E[R|\mathcal{D}_I] - \widehat{R}\right)^2$$

$$= \text{process variance} + \text{estimation error}$$

$\widehat{R} = E[R|\mathcal{D}_I]$ *"best estimate" of reserve*

Bayesian context – variance decomposition

$$\mathrm{Var}(R|\mathcal{D}_I) = E[\mathrm{Var}(R|\boldsymbol{\theta},\mathcal{D}_I)|\mathcal{D}_I] + \mathrm{Var}(E[R|\boldsymbol{\theta},\mathcal{D}_I]|\mathcal{D}_I)$$

$$= \text{average process variance} + \text{parameter estimation error.}$$

$\boldsymbol{\theta}$ *is model parameter vector modelled as random variable*

Claims reserving: Tweedie's compound Poisson model

$Y_{i,j}$ *are independent for* $i,j \in \{0,\ldots,I\}$

$$Y_{i,j} = 1_{\{N_{i,j}>0\}} \sum_{k=1}^{N_{i,j}} X_{i,j}^{(k)} \quad N_{i,j} \text{ and } X_{i,j}^{(k)} \text{ are independent}$$

$N_{i,j}$ *is Poisson distributed with parameter* $\lambda_{i,j}$

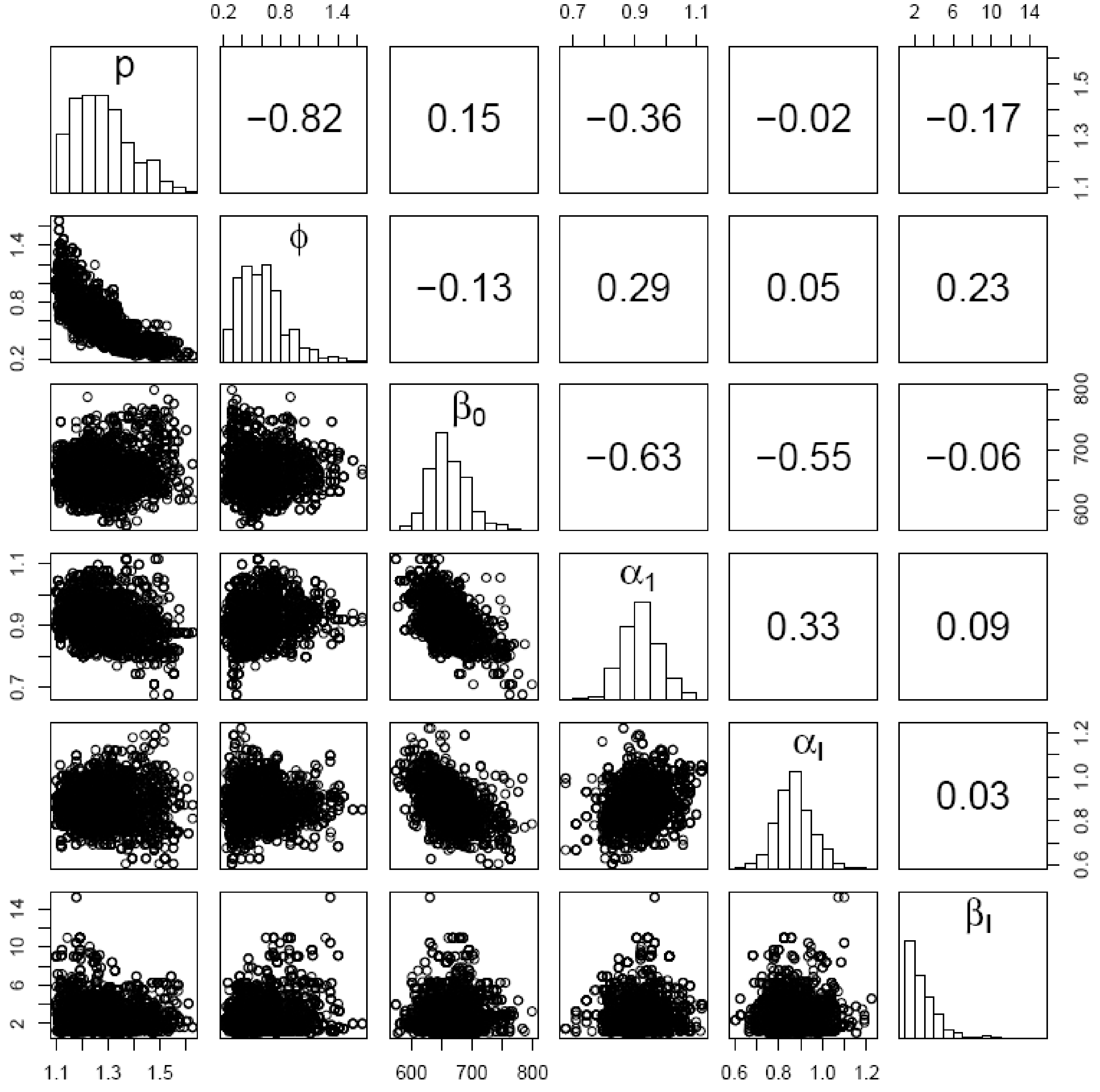
$X_{i,j}^{(k)}$ *are independent gamma severities with mean* $\tau_{i,j} > 0$ *and shape parameter* $\gamma > 0$

$$f_{\mu_{i,j}}(y;\phi_{i,j},p) = c(y;\phi_{i,j},p)\exp\left\{\phi_{i,j}^{-1}\left[y\,\frac{\mu_{i,j}^{1-p}}{1-p} - \frac{\mu_{i,j}^{2-p}}{2-p}\right]\right\}$$

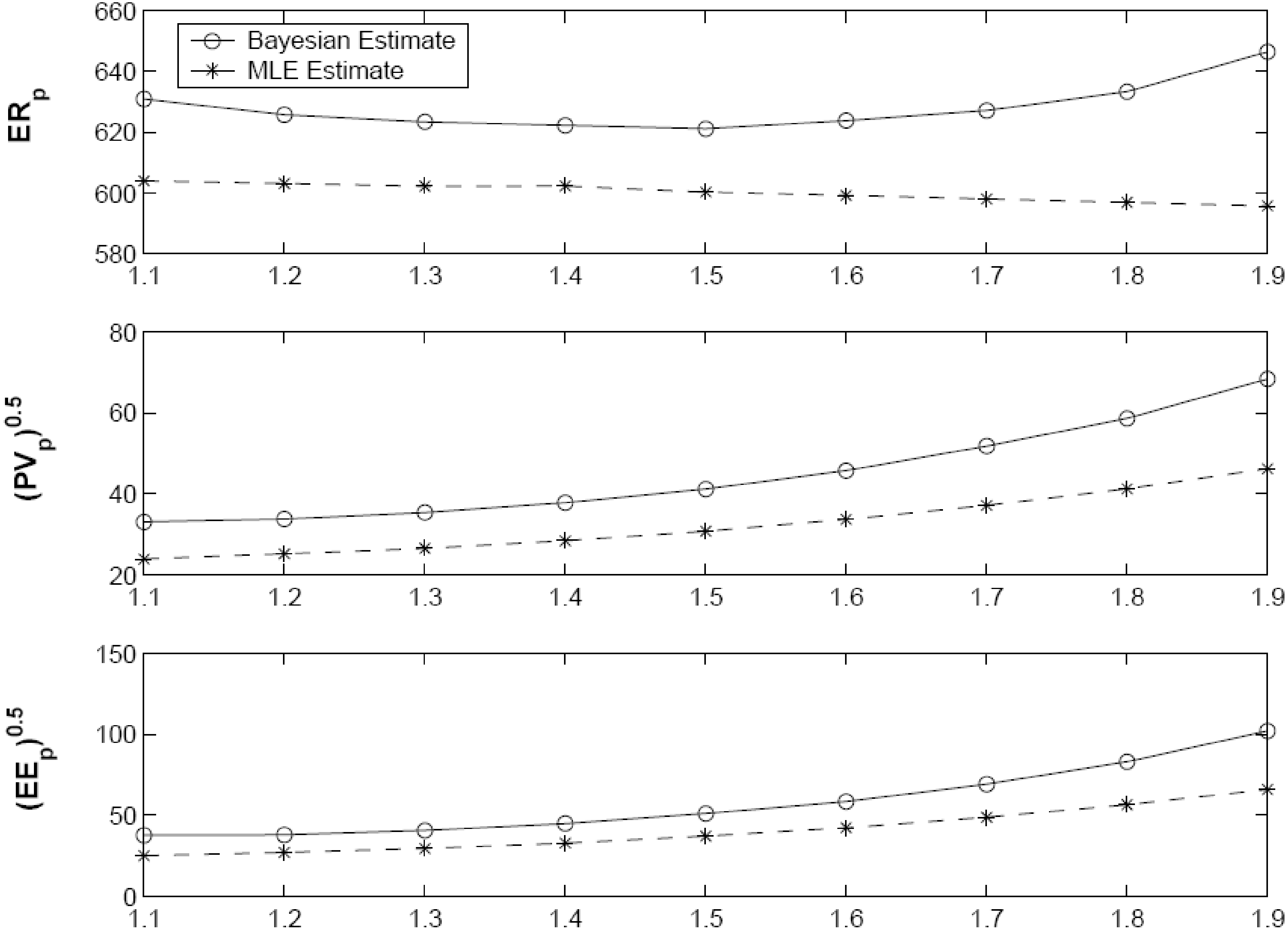
$$E\left[Y_{i,j}\right] = \frac{\partial}{\partial\theta_{i,j}}\kappa_p(\theta_{i,j}) = \kappa_p'(\theta_{i,j}) = \left[(1-p)\theta_{i,j}\right]^{1/(1-p)} = \mu_{i,j}$$

$$\mathrm{Var}\left(Y_{i,j}\right) = \phi_{i,j}\kappa_p''(\theta_{i,j}) = \phi_{i,j}\ \mu_{i,j}^{p}$$

$p \in (1,2)$, typically fixed by the modeller *Model Risk*


p
φ
β0
α1
αl
βl
-0.82
0.15
-0.36
-0.02
-0.17
-0.13
0.29
0.05
0.23
-0.63
-0.55
-0.06
0.33
0.09
0.03
0.2 0.8 1.4
0.7 0.9 1.1
2 6 10 14
1.1 1.3 1.5
600 700 800
0.6 0.8 1.0 1.2

Claims reserving – expected reserves (ER), process variance (PV), estimation error (ER). Bayesian (MCMC) vs Maximum Likelihood



Research topics for collaboration

- Combining different data sources: internal & external data with expert opinions (credibility theory, Bayesian techniques)
- Dependence between risks: copula methods, structural models
- Expert elicitation (Bayesian networks)
- Extreme Value Models (Modelling distribution tail)
- Efficient Markov Chain Monte Carlo, ABC MCMC, Monte Carlo, finite element/finite difference methods
- State-space models (Kalman/particle filters/SMC)

- Pattern recognition – trading strategies
- Stochastic/local volatility models
- Modelling operational/market/credit risks/insurance risks
- Pricing exotic options
- Modelling commodities, interest rates,
- Portfolio asset allocation

Recent journal publications

- **G. W. Peters, P.V. Shevchenko and M.V. Wüthrich** (2009). Chain Ladder Method: Bayesian Bootstrap versus Classical Bootstrap. Submitted to *Insurance: Mathematics and Economics*.
- **G. W. Peters, P. V. Shevchenko and M. V. Wüthrich** (2009). Model uncertainty in claims reserving within Tweedie's compound Poisson models. *ASTIN Bulletin* **39**.
- **P. V. Shevchenko** (2008). Implementing Basel II Loss Distribution Approach for operational Risk. Submitted to *Applied Stochastic Models in Business and Industry*.
- **Peters, G., P. Shevchenko and M.Wuthrich** (2009). Dynamic operational risk: modelling dependence and combining different data sources of information. *J. of Op Risk*
- **Shevchenko, P**. (2008). Estimation of Operational Risk Capital Charge under Parameter Uncertainty. *The Journal of Operational Risk* **3**(1), 51-63.
- **Luo, X., P. V. Shevchenko and J. Donnelly** (2007). *Addressing Impact of Truncation and Parameter Uncertainty on Operational Risk Estimates*. *The J. of Op. Risk* **2**(4), 3-26.
- **D. D. Lambrigger, P.V. Shevchenko and M. V. Wüthrich** (2007). The Quantification of Operational Risk using Internal Data, Relevant External Data and Expert Opinions. *J. of Op. Risk* **2**(3), 3-27.
- **Bühlmann,H., P. Shevchenko and M. Wüthrich**. *A "Toy" Model for Operational Risk Quantification using Credibility Theory. The J. of Operational Risk* **2**(1), 3-19, 2007.
- **Shevchenko, P. and M. Wüthrich** (2006). Structural Modelling of Operational Risk using Bayesian Inference: combining loss data with expert opinions. *J. of Op. Risk* **1**(3), 3-26.

www.cmis.csiro.au/Pavel.Shevchenko

CSIRO Mathematical & Information Sciences
Pavel Shevchenko

Email: Pavel.Shevchenko@csiro.au
www.cmis.csiro.au/Pavel.Shevchenko
www.csiro.au/people/Pavel.Shevchenko.html

www.csiro.au

Thank you

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